

The effect of Landau-Zener tunnelings on the nonlinear dynamics of cold atoms in a modulated laser field

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We study the motion of a cold atom in a frequency-modulated standing laser wave. If the detuning between the atomic electronic transition and the field is large, the atom moves in a modulated optical potential demonstrating known classical nonlinear effects such as chaos and nonlinear resonances. If the atom-field detuning is small, then two optical potentials emerge in the system, and the atom performs Landau-Zener (LZ) tunnelings between them. It is a radically non-classical behavior. However, we show that classical nonlinear structures in system's phase space (KAM-tori and chaotic layers) survive. Quantum effect of LZ tunnelings only induces small random jumps of trajectories between these structures (dynamical tunneling).

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I. INTRODUCTION

Cold atoms in a laser field is a popular system for the study of quantum chaos. Since 1970s, a lot of authors reported various nonlinear effects in semiclassical models of interaction between two-level atoms and laser waves (constant, modulated, and kicked): dynamical chaos [1, 2], dynamical localization [3–5], random walking of atoms [6], dynamical tunneling [7, 8], fractals [9], strange attractors [10], nonlinear resonances [11] etc. Some of these studies have not only theoretical but also methodological aspect of quantum-classical correspondence: they compare idealized nonlinear (in particular, chaotic) dynamics of a semiclassical model with real dynamics of a quantum system and analyze the difference.

As a rule, in order to observe nonlinear dynamical effects, the model must be simple enough. There are two popular approaches to observe clear nonlinear dynamics and chaos in atom-field models: (1) neglect the dynamics of atomic electronic transitions and study only mechanical motion of an atom in a modulated laser field (detuned far from optical resonance), (2) study the interaction between atomic electronical and mechanical degrees of freedom in a stationary field (near optical resonance).

First approach is less fundamental, because the system is non-autonomous. However, it requires comparatively simple experimental setup. Since 1990s, several important theoretical results on atomic chaos in a modulated field were directly tested in experiments. In particular, notable experiments were performed in order to observe dynamical localization [4] and dynamical tunneling [7]. The phenomenon of dynamical tunneling is very important in the framework of this paper, and we will discuss it in details in further paragraphs.

Second approach is more fundamental but methodologically more problematic because semiclassical approximation is not very good for the description of electronic transitions. We applied second approach in our early papers [9–11] using idealized semiclassical models and reported a lot of nonlinear effects. However, physical correctness

of these results was not clear. In [12, 13] more rigorous quantum analysis of atomic motion was performed without semiclassical approximation: atoms were considered as wave packets, not dot-like particles. This new model was used to test our old results. In a "chaotic" range of parameters of semiclassical models [9, 11], new model demonstrated random atomic Landau-Zener (LZ) transitions between two optical potentials. In [12], it was shown that LZ transitions (in quantum model) and dynamical chaos (in semiclassical model) produce similar statistical effect on atomic mechanical motion. However, subtle nonlinear effects in electronic transitions seem to disappear in quantum model.

In this paper we are trying to overcome limitations of both above-mentioned approaches and to study the nonlinear dynamics of an atom in a modulated standing wave near optical resonance. We use a stochastic trajectory model combining semiclassical dynamics (regular atomic motion far from standing-wave nodes) and random jumps (LZ transitions near the nodes). We study the dynamics by the method of Poincaré mapping (Poincaré section) and report the existence of prominent nonlinear structures (KAM-tori and chaotic areas) in systems's phase space even in presence of LZ tunnelings. The size of some nonlinear structures is large enough to overcome Heisenberg limitations. Therefore, they seem to be not just artifacts of our model but a real property of the system

We report not only the existence on nonlinear structures but also LZ-induced dynamical tunneling of atom between them. A pair of LZ tunnelings (from one potential to another and back) may cause a jump of atomic trajectory between two classically isolated nonlinear structures (e.g. two KAM-tori). Such quantum-induced jump is classically prohibited not by energy but by another constant of motion, and it is called dynamical tunneling. The phenomenon of dynamical tunneling was deeply studied in [7, 8] in the regime of large atom-field detunings without LZ transitions. We report similar effect in another physical situation: atom is close to optical resonance, and its dynamical tunnelings are directly caused

by pairs of successive LZ tunnelings.

II. EQUATIONS OF MOTION

In this paper we study the same system as in [15, 16]. A two-level atom with transition frequency ω_a and mass m_a moves in a standing laser wave with modulated frequency $\omega_f[t]$. We neglect spontaneous emission (excited state has a long lifetime, or some methods are used to suppress decoherence). In [15, 16], we started the analysis with Jaynes-Cummings Hamiltonian obtaining quantum equations of atomic motion. In this paper, however, we use a simplified stochastic trajectory model obtained in [16]. The model consists of semiclassical and stochastic parts: atom moves as a dot-like particle in a classical potential, but sometimes it performs random jumps.

Semiclassical part. Semiclassical approximation is valid when an atom moves between standing-wave nodes ($k_f X \neq \pi/2 + \pi n$, $\cos[k_f X] \neq 0$). This motion is regular and may be described by the Hamiltonian [13–16]

$$H = \frac{P^2}{2m_a} + \hbar\Omega U^\mp, \quad U^\mp = \mp \sqrt{\cos^2[k_f X] + \frac{(\omega_a - \omega_f[t])^2}{4\Omega^2}}. \quad (1)$$

Here Ω is a Rabi frequency (field intensity) and k_f is a wave vector. The sign of potential U^\mp is conserved, if the atom does not cross standing-wave nodes. Let us use the following normalized quantities: momentum $p \equiv P/\hbar k_f$, time $\tau \equiv \Omega t$, position $x \equiv k_f X$, mass $m \equiv m_a \Omega/\hbar k_f^2$ and detuning $\Delta[\tau] \equiv (\omega_f[\tau] - \omega_a)/\Omega$. Energy and potential (Fig. 1a, dashed lines) take simple forms

$$E = \frac{p^2}{2m} + U^\mp, \quad U^\mp = \mp \sqrt{\cos^2[x] + \frac{\Delta^2[\tau]}{4}}, \quad (2)$$

and we obtain semiclassical equations of motion

$$\dot{x} = \frac{p}{m}, \quad \dot{p} = -\text{grad}U^\mp = \frac{\sin[x] \cos[x]}{U^\mp}. \quad (3)$$

These equations were originally obtained in [14] for constant field ($U^\mp = U^\mp[x]$), but they also stay correct for modulated field ($U^\mp = U^\mp[x, \tau]$), if the modulation is slow [16]. Let us use the harmonic modulation

$$\Delta[\tau] = \Delta_0 + \Delta_1 \cos[\chi[\tau]], \quad \chi[\tau] \equiv \zeta\tau + \phi, \quad (4)$$

$$\zeta \ll 1, \quad \Delta_0 \sim \Delta_1 \ll 1.$$

Stochastic part. When an atom crosses wave nodes ($x = \pi/2 + \pi n$, $\cos[x] = 0$), the distance between potentials is minimal (Fig. 1a), and Landau-Zener (LZ) tunnelings between them occur with the probability [12]

$$W_{\text{LZ}} \simeq \exp \frac{-\Delta^2 m \pi}{4|p_{\text{node}}|}, \quad (5)$$

where p_{node} is an atomic momentum at the moment of node crossing. For example, for $|\Delta| \ll 1$, $x[0] = 0$, $U[0] = U^-$, it may be approximately estimated as [14]

$$p_{\text{node}} \simeq \pm \sqrt{p^2[0] - 2m}. \quad (6)$$

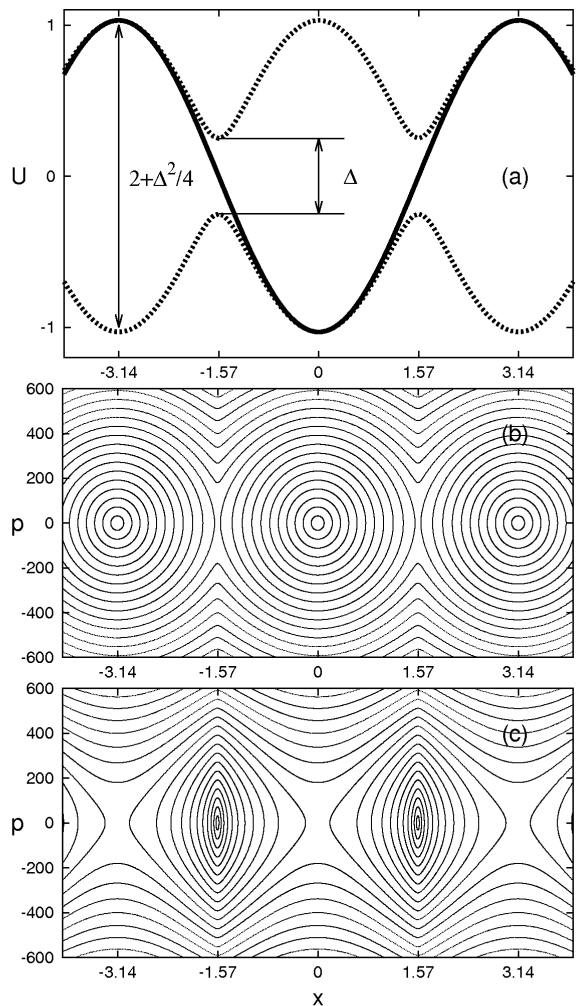


FIG. 1: Optical potentials and their phase portraits: (a) dashed line: potentials U^\pm , solid line: standing wave $-\cos[x]$, (b) phase portrait for U^- , (c) phase portrait for U^+

If LZ tunneling occurs, the potential U^\mp changes its sign and the atom continues its motion.

As a summary, a dot-like atom moves according to the equations (3), but sometimes (when $\cos[x] = 0$) the potential U^\mp changes its sign with the probability (5).

III. PHASE SPACE STRUCTURE IN LIMIT CASES OF STATIONARY FIELD AND LARGE DETUNINGS

In a stationary field ($\Delta_1 = 0$), atomic motion is simple. Atom moves in a 2D phase space: its state is comprehensively given by its position and momentum. In Fig. 1b and c, phase portraits for potentials U^\mp are shown. In order to compute them, we simulated a series of trajectories with different initial positions ($x[0] = 0, \pm\pi$) and momenta (a lot of values) and draw them in x, p

planes. All phase trajectories are periodic: slow atoms oscillate in potential wells (closed trajectories) and fast atoms move ballistically with oscillating momentum. Far from resonance ($|\Delta| \gtrsim 1$) the sign of potential is constant, so an atom moves along the same trajectory during the evolution. Near resonance ($|\Delta| \ll 1$) LZ transitions between U^- and U^+ become possible, so the structure of phase space may switch spontaneously between Fig. 1b and Fig. 1c. This may produce random walk of an atom (in terms of stochastic trajectory model) and splittings of atomic wave packets (in quantum terms) [12–14]. However, this does not produce any important nonlinear effects such as dynamical chaos.

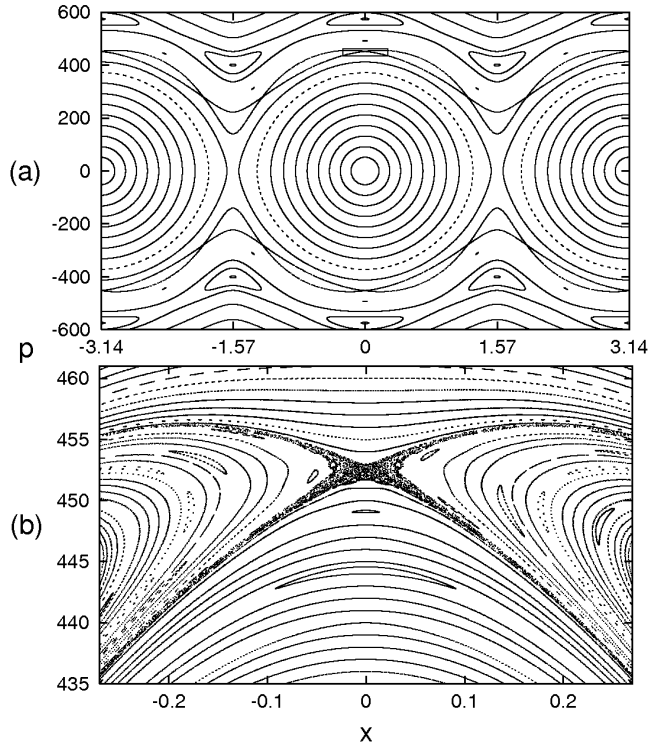


FIG. 2: Poincaré mappings in x, p plane for large detuning ($\Delta_0 = -0.3$): (a) general view, (b) magnified fragment with chaotic layer and regular curves

Presence of field modulation (4) ($\Delta_1 \neq 0$) radically changes the dynamics. System's phase space becomes 3D because the phase of modulation $\chi[\tau]$ may be regarded as a third variable. Far from resonance ($|\Delta| \gtrsim 1$) atom demonstrates typical dynamics of nonlinear pendulum with periodical modulation of parameters. Similar pendulum regimes were studied in countless physical systems [17]. In order to demonstrate the structure phase space, let us use the method of Poincaré mapping on the plane x, p . The idea of Poincaré mapping (Poincaré section) is to draw not the whole trajectory but only its isolated points taken periodically after each cycle of modulation. To be concrete, let us take points when $\chi = 0$.

In Fig. 2, we put $\Delta_0 = -0.3$, $\Delta_1 = -0.1$, $\zeta = 0.02$ (Δ oscillates in a range $-0.4 \leq \Delta \leq -0.2$, so the probability of LZ transitions is negligible ($W_{LZ} \lesssim \exp[-10]$)) and compute Poincaré sections for a set of trajectories with different initial atomic momenta and positions in potential U^- . Most of them look similar to 2D-trajectories shown in Fig. 1b, but some others have sophisticated form. This picture is rather typical with chaotic Hamiltonian systems [17, 18]. We see various regular Kolmogorov-Arnold-Moser (KAM) invariant curves (produced by nonlinear resonances of different order) and chaotic (stochastic) layers. In this figure, the chaotic layers are narrow and the majority of trajectories are regular. In Fig. 2b we magnify a small area of Fig. 2a showing a saddle-like region of chaotic motion between regular curves.

Let us note that in non-autonomous system (3), energy (2) is not conserved. However, for some values of initial conditions, system contains another integral of motion (having no general analytical form), so the effective available phase space for particle's motion is a sophisticated 2D surface named KAM-tori (KAM-curves are the sections of these tori by the plane $\chi = 0$). A particle moves periodically or quasiperiodically on one of these surfaces. On the other hand, for some other initial conditions no integrals of motion exist, so the effective available phase space is 3D and particle's motion is chaotic. The transition of particle from one KAM-tori to another (or to chaotic layer) is prohibited by the integral of motion.

IV. PHASE SPACE STRUCTURE IN PRESENCE OF MODULATION AND LANDAU-ZENER TRANSITIONS

It is not surprising to see KAM-tori in a phase space of simple perturbed nonlinear pendulum. More interesting picture is observed for smaller detunings in presence of LZ transitions. In Fig. 3 we built Poincaré sections for system (3) with the parameters $\Delta_0 = -0.2$, $\Delta_1 = -0.1$, $\zeta = 0.02$. Now Δ oscillates in a range $-0.3 \leq \Delta \leq -0.1$, so $W_{LZ} \lesssim \exp[-2]$, and LZ tunnelings occur many times during the evolution. The surprising fact is that KAM-structures still exist. They are not destroyed by random events of LZ tunnelings. However, the dynamics has two new important features.

First feature is that during the evolution, the potential switches between U^+ and U^- . Each potential corresponds to its own structure of phase space. Therefore, in Fig. 3a we observe two sets of overlapping structures. First set (for U^-) is similar to Fig. 2a (although chaotic regions are larger), while the second one is a perturbed version of Fig. 1c. For any single trajectory in Fig. 3a, the motion in U^+ is comparatively slow ($|p| \lesssim p_{\text{node}}$) while the motion in U^- is comparatively fast ($|p| \gtrsim p_{\text{node}}$). However, different trajectories correspond to different values of p_{node} (6). Therefore, it is better to study separate pictures for each trajectory. In Fig. 3b and c we present Poincaré mappings for two particular trajectories from

Fig. 3a. Here, it is easy to see structures corresponding to U^+ and U^- without significant overlapping.

Second feature is that a single atomic trajectory may visit many separate KAM-curves and chaotic regions. In absence of LZ transitions rich structures shown in Fig. 3b, c and d can be produced only by ensembles of trajectories with different initial momenta. In particular, in Fig. 3d we see a fine structure of chaotic layer and regular curves. In a classical system the transition between them is prohibited. However, in presense of LZ transitions they are all produced by a single atomic trajectory. This is because time periods of atomic motion between successive node crossings in U^+ and U^- potentials are different, and none of them (in general case) is synchronized with field modulation. Therefore, after two successive LZ transitions atom returns to the same potential, but with another (in fact, random) values of χ and Δ . If some integral of motion existed for this trajectory, it changes its value, and the motion continues along new KAM-curve. In other words, a pair of LZ tunnelings produces a dynamical tunneling [7, 8] of an atomic trajectory between classically isolated areas of phase space

V. DISCUSSION AND CONCLUSION

Two-level atom in a far-detuned modulated laser field is a popular physical system for observation of nonlinear effects such as chaos and nonlinear resonance. Its phase space contains KAM-tori and chaotic layers typical for perturbed pendulum. On the other hand, if an atom is close to optical resonance, it performs Landau-Zener (LZ)

tunnelings between two optical potentials, and this is not a classical pendulum behavior. In this paper, however, we have shown that LZ tunnelings, being a quantum random effect, do not destroy classical nonlinear structures in phase space. They affect only particular realizations of atomic motion producing a phenomenon of dynamical tunneling [7, 8]. Large atom-field detuning (used in famous works [2–5, 7, 8]) is not necessary for observation of nonlinear dynamics of atoms in a modulated field.

These results were obtained with the use of stochastic trajectory model simplifying real quantum dynamics of atoms. However, it seems to be correct in the framework of our study. We avoided to analyze phase-space structures prohibited by Heisenberg relation. All coordinate scales were of the order of optical wavelength (hundreds of nanometers), and all atomic momentums were much larger than the photon momentum. In [16] we presented a quantitative comparison between this stochastic trajectory model and purely quantum model, and the correspondence was good (the values of variables and parameters was similar to those used in this paper).

The perspective of experimental test of our result is not a simple issue. A huge number of precise experiments with single atoms is needed. Each atom must be prepared in appropriate initial state and its momentum and position must be measured after exact integer number of field modulation periods. In future studies we are going to simulate such experiment numerically (using quantum equations), but this will take a huge computational time.

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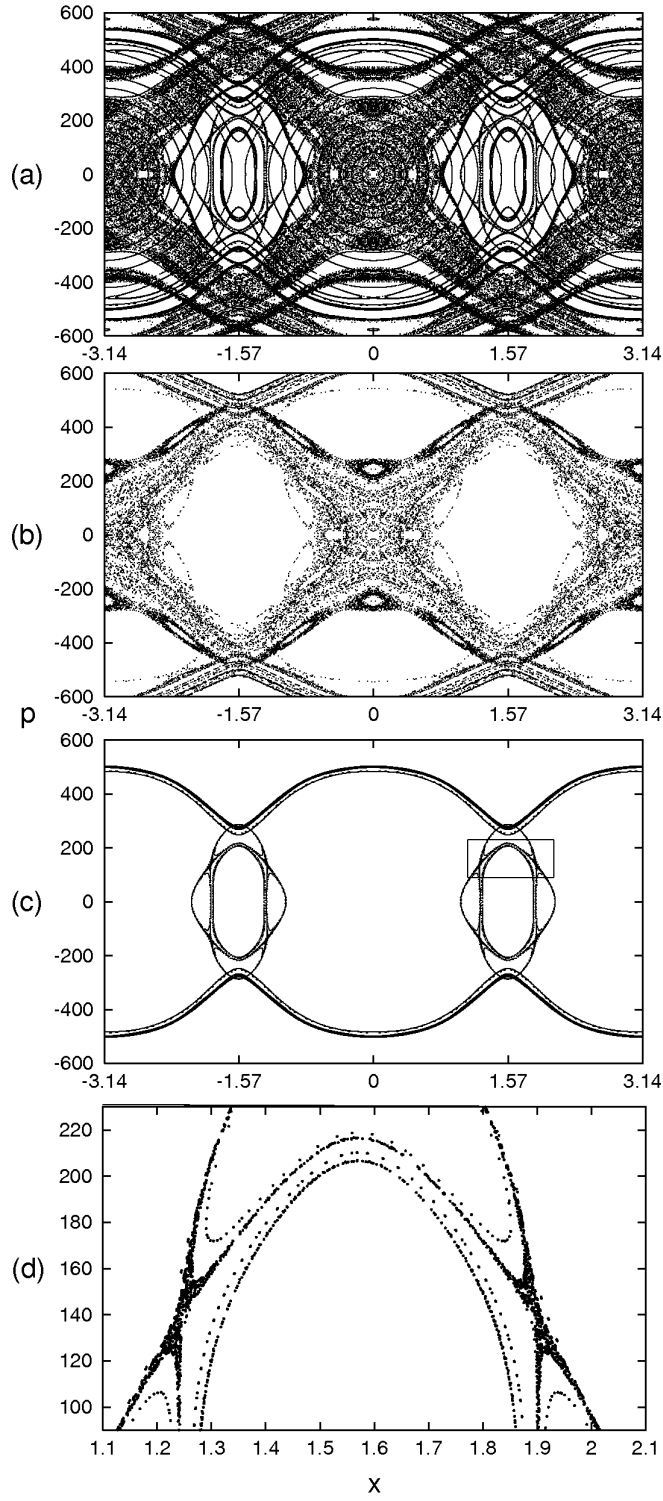


FIG. 3: Poincaré mappings in x, p plane for small detuning ($\Delta_0 = -0.1$): (a) general view (many trajectories), (b) single chaotic trajectory with $p[0] = 660$, (c) single trajectory with regular and slightly chaotic parts with $p[0] = 460$, (d) magnified fragment of (c) showing chaotic layer and neighbour regular curves visited by a single trajectory